Esercizio 1314
(File scaricato da http://www.extrabyte.info)

Determinare il differenziale totale del second'ordine della funzione:

$$f\left(u,v\right)=u^{v},$$

dove:

$$u\left(x\right)=ax,\ \ v\left(y\right)=by$$

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Soluzione
Abbiamo la funzione composta $f\left[u\left(x\right),v\left(y\right)\right]$, quindi:

$$df=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy$$

Calcoliamo le derivate parziali rispetto a x,y:

$$\begin{aligned}\frac{\partial f}{\partial x}&=\frac{\partial f}{\partial u}\frac{du}{dx}=a\frac{\partial f}{\partial u}\\\frac{\partial f}{\partial y}&=\frac{\partial f}{\partial v}\frac{\partial v}{\partial y}=b\frac{\partial f}{\partial v}\end{aligned}$$

Quindi:

$$\begin{aligned}df&=a\frac{\partial f}{\partial u}dx+b\frac{\partial f}{\partial v}dy\\&=af_{u}\left[u\left(x\right),v\left(y\right)\right]dx+bf_{v}\left[u\left(x\right),v\left(y\right)\right]dy\end{aligned}$$

Calcoliamoa $d^{2}f$

$$\begin{aligned}d^{2}f&=d\left(df\right)\\&=adf_{u}dx+bdf_{v}dy\\&=a\left(a\frac{\partial^{2}f}{\partial u^{2}}dx+b\frac{\partial^{2}f}{\partial u\partial v}dy\right)dx+b\left(a\frac{\partial^{2}f}{\partial v\partial u}dx+b\frac{\partial^{2}f}{\partial v^{2}}dy\right)dy\\&=a^{2}\frac{\partial^{2}f}{\partial u^{2}}dx^{2}+2ab\frac{\partial^{2}f}{\partial u\partial v}dxdy+b^{2}\frac{\partial^{2}f}{\partial v^{2}}dy^{2}\end{aligned}$$