Esercizio 1253
(File scaricato da http://www.extrabyte.info)

Sia:

$$f : U \longrightarrow \mathbb{R}$$
$$(u, v) \longrightarrow f(u, v) \in \mathbb{R}, \ \ \forall (u, v) \in U,$$

con $U \subseteq \mathbb{R}^2 \mid U$ è un campo. Per ipotesi:

$$u = u(x, y), \ v = v(x, y),$$
$$u(x, y), v(x, y) \ \text{parzialmente derivabili in } \mathbb{R}^2$$

donde la funzione composta:

$$F(x, y) \equiv f[u(x, y), v(x, y)]$$

Determinare 1) le derivate parziali $\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}$; 2) esplicitare tali espressioni se: $u(x, y) = x^2 - y^2$, $v(x, y) = e^{xy}$.

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Soluzione

$$\frac{\partial F}{\partial x} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial x}$$
$$\frac{\partial F}{\partial y} = \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y}$$

Se: $u(x, y) = x^2 - y^2$, $v(x, y) = e^{xy}$

$$\frac{\partial F}{\partial x} = 2x\frac{\partial f}{\partial u} + ye^{xy}\frac{\partial f}{\partial v}$$
$$\frac{\partial F}{\partial y} = -2y\frac{\partial f}{\partial u} + xe^{xy}\frac{\partial f}{\partial v}$$