Esercizio 1004
(File scaricato da http://www.extrabyte.info)

Calcolare i seguenti integrali:

$$\begin{gathered}\int \frac{dx}{x^2+2x+5} \\ \int \frac{2dx}{3x^2-2x+4}\end{gathered} \tag{1}$$

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Soluzione
Scriviamo:

$$x^2+2x+5 = x^2+2x+4+1 = (x+2)^2+1$$

Quindi:

$$\begin{aligned}\int \frac{dx}{x^2+2x+5} &= \int \frac{dx}{(x+2)^2+1} \\ &= \int \frac{d(x+2)}{(x+2)^2+1} \\ &= \arctan(x+2)+C\end{aligned}$$

Passiamo al secondo integrale:

$$\begin{aligned}3x^2-2x+4 &= 3(x+k)^2+l \\ &= 3x^2+6kx+3k^2+l \\ &\Longrightarrow \begin{cases} k=-\frac{1}{3} \\ l=\frac{11}{3}\end{cases} \\ &\Longrightarrow 3x^2-2x+4 = \frac{11}{3}\left[\left(\frac{3x-1}{\sqrt{11}}\right)^2+1\right]\end{aligned}$$

Quindi:

$$\begin{aligned}\int \frac{2dx}{3x^2-2x+4} &= \frac{6}{11}\int \frac{dx}{\left(\frac{3x-1}{\sqrt{11}}\right)^2+1} \\ &= \frac{6}{\sqrt{11}}\int \frac{d\left(\frac{3x-1}{\sqrt{11}}\right)}{\left(\frac{3x-1}{\sqrt{11}}\right)^2+1} \\ &= \frac{6}{\sqrt{11}}\arctan\left(\frac{3x-1}{\sqrt{11}}\right)+C\end{aligned}$$